CHAPTER 7

Probing the neural basis of rational numbers: The role of inhibitory control and magnitude representations

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Rational numbers beyond the classroom

Rational numbers are a notoriously challenging part of the elementary mathematics sequence (National Mathematics Advisory Panel, 2008). Less acknowledged is the role they play beyond the classroom or the job site (Handel, 2016; Matthes, Christoph, Janik, & Ruland, 2014), in areas like physical health and personal finance. For example, studies of health numeracy have found that items involving rational numbers are particularly challenging for patients—56% of diabetics could not correctly compute 18 g × 3.5 (servings) to determine the number of carbohydrates in a snack package (Cavanaugh et al., 2008)—making them strong predictors of health outcomes (Chakkalakal et al., 2017; Osborn et al., 2013). While millions lost their homes during the great recession, many did not. What differentiated these borrowers was not their incomes or credit histories, but their ability to compute with rational numbers (Gerardi, Goette, & Meier, 2013). These results indicate the far-reaching effects of rational number difficulties but also highlight the potential for insights from math cognition to have far-ranging benefits. But doing so requires understanding, both in terms of brains and behavior, what makes rational numbers so difficult for so many learners.

Deconstructing "whole number bias"

Like other mathematical content domains, rational numbers introduce new notation—numerator, denominator, decimal point, percent symbol—that connotes new mathematical concepts. But in contrast to domains like geometry,

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which formalize learners' previous experiences with shapes and angles, rational number understanding builds on, but can also contradict, learners' preexisting knowledge of the whole number system. Learners often incorrectly apply whole number principles and properties when processing rational numbers, a phenomenon that has been termed *whole number bias*[a] (Ni & Zhou, 2005).

While it may be tempting to attribute difficulties with rational numbers to simply overextending whole number properties, this stance oversimplifies the complicated relationship between whole and rational numbers. Table 1 identifies three categories of whole number properties that must be considered in order to fully understand difficulties that arise when learning about rational numbers: First are properties that never apply to rationals, such as, whole numbers have unique symbols and successors, while rational numbers do not (Category 1). Second are properties that always apply for whole numbers but only sometimes apply to rationals. For instance, more digits always connotes a larger quantity for whole numbers ($27 > 8$), but not so for all rational numbers ($0.27 < 0.8$) (Category 2). Finally, there are properties of whole numbers that also apply to rationals, although this fact may not be initially apparent by learners. In this category are deep properties of real numbers, such as all numbers represent magnitudes and can be uniquely ordered (Category 3). Indeed, initially learners may only have a superficial understanding that whole numbers have these properties, and mastering Category 3 for rationals may also induce conceptual change for whole numbers (De Smedt, this volume).

Crucially, these categories are not isolated but rather interact in challenging ways for learners. Take the case of symbolic representations: each whole number quantity can be specified uniquely by one (and only one) symbol,[b] but for rational numbers, there are infinite ways to denote a rational quantity. A "half" can be written as 1/2, 2/4, 5/10, 50%, 0.5, 0.500, …, whereas there is only one symbolic expression for the quantity "two," namely, "2" (Braithwaite & Siegler, 2018). These multiple representations decouple previously learnt

[a]The terms *whole* and *natural number bias* have both been used to describe this phenomenon. Based on the classification of natural numbers as positive integers (1, 2, 3, …) and whole numbers as positive integers and zero (0, 1, 2, 3, …), and the observation that some properties which interfere with rational number processing are related to the role of zero, we use the term *whole number bias* here.

[b]Whole numbers are a subset of rational numbers. Therefore, within the rational number system there are multiple representations for whole number quantities ($2 = 4/2 = 200\% = 2.00 = \ldots$). However, within the whole number system, symbols are unique, thus the only whole number symbol that represents the quantity "two," is "2."

Table 1 Examples of properties of whole numbers and their status for rational numbers.

Category	Property	Whole numbers	Rational numbers
1	Unique symbol Finite number in interval Unique successor	✔ ✔ ✔	✘ ✘ ✘
2	More digits → larger number Larger numerals → larger number Multiply → larger number	✔ ✔ ✔	? ? ?
3	Have magnitude Uniquely ordered	✔ ✔	✔ ✔

Category 1 properties never apply to rational numbers. Category 2 properties sometimes apply to rational numbers and sometimes do not. Category 3 properties always apply to rational numbers.

associations between number symbols and their magnitude, which results in Category 2 properties, like more digits can represent a smaller rational number (Huber, Klein, Willmes, Nuerk, & Moeller, 2014; Varma & Karl, 2013). Finally, multiple representations may obscure the fact that rational numbers represent quantities and can be uniquely ordered just like whole numbers. Thus 1/2 is less 52%, but more than 0.2. In fact, in this last case, "whole number bias" would actually be helpful, as these properties of whole numbers should be extended to rational numbers. Yet, many individuals struggle to understand that rational numbers refer to numerical quantities.

In this chapter, I argue that deep understanding of rational numbers requires mastering properties from all three of the categories summarized in Table 1. To do so, learners must *expand* and *refine* their understanding of numbers. In mastering Categories 1 and 2, learners must overcome their prior whole number knowledge, a task made more difficult because this knowledge is often acquired implicitly. I propose that the key cognitive capacity that supports refining numerical representations is inhibitory control, the ability to resolve interference from competing information. In mastering Category 3 properties, learners must expand their representation of whole numbers to include rational numbers. I propose that the key cognitive capacity that supports expansion is developing a magnitude representation of rational number quantities.

In this chapter, I begin by highlighting the role implicit understanding may play in rational number understanding and then review behavioral studies examining the role of inhibitory control in rational number processing using individual differences methods. Next, I consider magnitude representation of rational numbers in both symbolic and nonsymbolic contexts, highlighting the challenges in creating stimuli to independently assess whole and rational number influences. Next, the handful of neuroimaging studies which have examined rational number processing are summarized and situated with respect to symbolic and nonsymbolic whole number imaging results. Finally, I offer suggestions and recommendations for future work to leverage our growing understanding of rational numbers to develop brain-informed instruction for this vital, but challenging, class of numbers.

Implicit understanding of number systems

A growing body of research examines how mathematical content can be learned implicitly, through statistical regularities, rather than from explicit instruction. While implicit understanding may be a hallmark of expertise (Weber, 2001), it can also result in overgeneralization and misconceptions (Chang, Koedinger, & Lovett, 2003). For example, when learning to count, young children often assume that they must proceed from left to right, absorbing this nonessential feature because all examples they have seen follow that order (Gelman & Gallistel, 1978). Another example, which illustrates the role instruction and context can play in implicit learning, comes from the phenomenon of misinterpreting the equal sign as a directive to complete an operation. In elementary school, instruction emphasizing the operational meaning further reinforces this misinterpretation (Baroody & Ginsburg, 1983). Yet, in middle school varying the prior context (i.e., $4+8+5+4=\_\_\_$ vs $4+8+5=4+\_\_\_$) can lead either to the operational misinterpretation or to the correct interpretation as a symbol of equivalence (McNeil & Alibali, 2005). These latter results suggest both that instruction can lead to systematic misunderstanding, but also that context can implicitly activate appropriate (or inappropriate) representations in mathematical domains.

A similar phenomenon may be happening in rational number instruction. For example, early emphasis on mastering the count sequence may inadvertently render understanding the concept of fractional numbers more difficult. Similarly, students may have already noticed that numbers with more digits are larger than numbers with fewer digits, well before learning the place value system in early elementary school (National Governors Association Center for Best Practices and Council of Chief State School Officers, 2010). Although I know of no formal research on this topic, informal discussions with educators confirm that in typical instruction, the introduction of rational numbers does not explicitly highlight properties of these numbers that contradict students' prior implicit knowledge. Illustratively, one math reference work for middle-grade teachers (Kaplan and Great Source Education Group, 2003) uses only consistent examples in the sections on comparing and ordering decimals (sections 018–020). Even the "Math Alert," which highlights common mistakes, focuses on the fact that lower numbers can be better in timed sporting events, and no mention is made of the possibility that more digits can connote a smaller number.

While many sources have noted the challenge for learners presented by differences between rational and whole numbers properties (Carey, 2011; Obersteiner, Dresler, Bieck, & Moeller, 2019), less acknowledged is the distinction between Category 1 properties, where whole number understanding is never appropriate, and Category 2 where, in some instances, correct responses are compatible with whole number properties. For example: 0.87 is larger than 0.2, a conclusion which could be erroneously arrived at by noting that 87 is larger than 2. That is, one's prior knowledge of whole numbers would actually lead to the correct answer in many instances. While the distinction between Category 1 and Category 2 is rarely stated explicitly, it forms the basis of most experimental designs in this domain, which specifically balance the number of problems that are congruent or incongruent with whole number knowledge (Obersteiner, Van Dooren, Van Hoof, & Verschaffel, 2013; Vamvakoussi, Van Dooren, & Verschaffel, 2012; Varma & Karl, 2013).

From the implicit, or statistical learning, point of view, these designs present an additional hurdle: prior context may be influencing the performance in the current situation. Negative priming studies illustrate the effects of context in moving between whole and rational representations. Studies in these domains employ a cue trial, which is thought to activate a particular representation, and then a probe trial is presented to assess the effects of the cue on subsequent performance. In the case of decimals, Roell and colleagues (Roell, Viarouge, Houde, & Borst, 2017, 2019) have found that comparing decimal pairs incongruent with whole number knowledge reduces performance on a subsequent congruent trial, relative to a neutral cue trial.

Beyond the laboratory, the majority of one's everyday numerical experiences involve whole numbers, and modern cultural practices further reduce real-world opportunities to process rational numbers with features incongruent with whole number information. The adoption of the decimal system for money, with two and only two significant digits—as recently as 1971 in the United Kingdom (Lee, 2010)—essentially removes the possibility for this confusion in the currency system. Likewise, metrification usually replaces fractional systems (Vera, 2012), and the flexibility of the International System of Units (SI) means decimal quantities can easily be replaced by whole numbers (0.6 kg = 600 g). One notable exception, the Library of Congress Classification system, still uses both whole and decimal numbers, a consistent source of confusion for university library users (Murphy, Long, & MacDonald, 2013). Future work is needed to assess the ways that both shorter (contextual) and longer term (cultural) exposure to whole numbers (and compatible rational numbers) influences rational number processing.

Behavioral evidence for the role of inhibitory control in rational number processing

Highlighting the role of implicit knowledge in understanding number systems motivates the necessity of inhibitory control in mastering rational numbers. Information that is learned implicitly and hence activated automatically may be especially difficult to override. This contention meshes with the robust body of literature demonstrating performance deficits on rational number tasks when correct responses conflict with whole number knowledge, in children (Meert, Gregoire, & Noel, 2009, 2010) and adults (Bonato, Fabbri, Umiltà, & Zorzi, 2007; DeWolf & Vosniadou, 2015; Huber et al., 2014; Varma & Karl, 2013). Yet, to establish that these effects are driven by the need for inhibitory control requires more than reporting impairment on incongruent cases. Individual differences designs leverage natural variation in inhibitory control across participants to test the need for this capacity in rational number processing. Far fewer studies have employed this approach, as the inclusion of nonmathematical tasks introduces several potential pitfalls. In this section, I review individual differences studies examining the connection between inhibitory control and rational number processing, with reference to two crucial methodological issues: how to (1) measure inhibitory control and (2) account for the contributions of other executive functions.

Measuring inhibitory control

The individual differences design requires the selection of measures to capture the cognitive construct of inhibitory control. Unfortunately, at the most basic level, the field lacks consensus about how best to measure inhibitory control (Lee & Lee, 2019), or even how to organize the myriad measures of inhibition. Several theoretical accounts do not treat the construct as unitary, instead dividing it into two or more forms (Friedman & Miyake, 2004; Kipp, 2005). Here, following Avgerinou and Tolmie (2019), I distinguish response inhibition (the inhibition of prepotent responses) from semantic inhibition (the inhibition of prepotent knowledge). Under this view, tasks like Go/No Go or Stop-Signal that set up a particular response which must be inhibited on some trials constitute response inhibition, whereas tasks which involve inhibiting responses which are favored by prior knowledge, like color-word and numerical Stroop, are considered measures of semantic inhibition. This division contrasts with the organization of Friedman and Miyake (2004), which considered both Stop-Signal and color-word Stroop as examples of prepotent response inhibition, which they contrasted with resistance to distractor interference. The division between behavioral and cognitive inhibition (Harnishfeger, 1995) is more akin to the ideas presented here, as this view distinguishes between inhibiting actions (i.e., motor responses, impulse control) from the active suppression of the contents of working memory (Kipp, 2005). However, the latter case often includes within cognitive inhibition, episodic memory paradigms, such as directed forgetting (Harnishfeger & Pope, 1996), where the information that must be inhibited was only recently and briefly presented. By contrast, in semantic inhibition the information that must be inhibited has been acquired over multiple, prolonged exposures and is thus stored in long-term memory (see Borst, this volume).

In the Go/No Go task, a classic measure of response inhibition, participants are presented with a stream of stimuli, where one stimulus is presented often (80% of the time) and the other rarely (20% of the time) and participants must respond to the frequent stimuli but withhold responses to the rare stimuli, setting up a propensity to respond. In the Animal Stroop task (Merkley, Thompson, & Scerif, 2015), a more recently developed measure of semantic inhibition, participants must identify the larger of two animals in real life, while the physical size of the presented animals is manipulated (e.g., a mouse drawn larger than an elephant). Avgerinou and Tolmie (2019) used these tasks to examine the contributions of both forms of inhibition to the processing of intuitive and counterintuitive fractions and decimal problems, in 8–10-year old children. They also assessed the effects of cognitive load, by manipulating how easily relevant information could be gleaned from a short-word problem. As expected, participants were slower and less accurate when the fraction and decimal quantities contradicted prior whole number knowledge (larger denominator indicating smaller fraction, more digits indicate smaller decimal). Although they did not provide first-order correlations, they do report regression analyses, showing response inhibition predicting counterintuitive performance under moderate cognitive load, and both response and semantic inhibition predicting under high-load conditions. These results confirm the need for inhibitory control in processing rational numbers, especially in more naturalistic settings where learners must seek information within a text display.

A common semantic inhibition measure in studies of numerical and mathematical cognition is the numerical Stroop task (Henik & Tzelgov, 1982), where single-digit numbers are compared, while the physical size of the numerals is manipulated to be either congruent (3 vs 7) or incongruent (3 vs 7) with numerical quantity. Recently, Gómez, Jiménez, Bobadilla, Reyes, and Dartnell (2015) used this task to assess relations between inhibitory control and rational number understanding, as measured by a fraction comparison task. They found that better performance on the numerical Stroop task was related to better performance on the fraction comparison task, especially in the incongruent cases. However, they also reported that math skills (as measured by in-school testing) explained this relationship. The authors interpret their results to imply that math abilities support both inhibitory control and rational number knowledge, and hence there is no direct link between inhibition and fraction understanding.

An alternative interpretation of these results is possible. First, note that the numerical Stroop task is a domain-specific measure of inhibitory control, as it measures inhibition between physical size and numerical magnitude. Thus those who excel at this task could have better understanding of numerical magnitude, which would support both fraction comparison and mathematical skills (Schneider et al., 2017; Ye et al., 2016). Without a domain general measure, one cannot assess the pure contribution of inhibitory control to fraction comparison. Second, these results could be equally consistent with the proposal that inhibitory control contributes to math skills, not just directly, but also by supporting the acquisition of fraction magnitude knowledge, which in turn supports other skills, like rational number arithmetic (Coulanges et al., 2021). Longitudinal studies in children support this conceptual trajectory, as students first acquire fraction magnitude understanding before mastering fraction arithmetic skills (Kainulainen, McMullen, & Lehtinen, 2017; Van Hoof, Degrande, Ceulemans, Verschaffel, & Van Dooren, 2018). Unfortunately, no studies to date have considered the effect of inhibitory control in the progression toward more sophisticated rational number understanding.

Another classic measure of inhibitory control is the Flanker task (Eriksen & Eriksen, 1974), where participants must respond according to the middle stimuli, ignoring the flanking stimuli. In modern formulations, a centrally presented stimulus points to the left or right and is surrounded by either congruent (e.g., <<<<<) or incongruent (e.g., >><>>) stimuli. While this task fits generally under the category of response inhibition in the current scheme, as the stimuli itself sets up the inhibition context, it can also be considered a form of visual distractor inhibition task (Friedman & Miyake, 2004; Lee & Lee, 2019). Matthews, Lewis, and Hubbard (2016) used this task to control for the potential role of inhibitory control in the relationship of nonsymbolic ratio comparison with symbolic fraction understanding and general math skills. They report that their key result—nonsymbolic fraction acuity predicts performance on symbolic measures—was robust to the inclusion of inhibitory control. In fact, zero-order correlations revealed that Flanker task performance was not significantly related to any of their outcome measures (number line estimation of rational numbers, symbolic fraction knowledge, or students' college entrance exam score). These null results suggest that visual distractor inhibition measures, like the Flanker task, may not be capturing the relevant aspect of inhibition required by rational number processing. More broadly, individual differences studies suggest that measures of semantic inhibition carry more predictive power than response inhibition measures.

Accounting for other executive functions

Studies on the specific role for inhibitory control in rational number processing play out within the broader context of research examining relations between executive functions (EF) and academic achievement. Here, I take the tripartite view of executive functioning as comprising (1) working memory, the ability to maintain and manipulate information; (2) inhibitory control, the ability to resolve interference from competing information; and (3) cognitive flexibility, the ability to switch between cognitive modes (Diamond, 2013). Among the three, the majority of work has focused on the relationship of math outcomes with working memory (Peng, Namkung, Barnes, & Sun, 2016), while fewer studies have examined on the role of inhibitory control and cognitive flexibility, also called shifting (Friso-van den Bos, van der Ven, Kroesbergen, & van Luit, 2013). In the realm of rational numbers, the focus thus far has been on working memory, either in terms of its direct relations (Fuchs et al., 2014; Hansen et al., 2015) or as a control measure when relating fraction understanding to later academic achievement (Siegler et al., 2012).

Robust relations are found between all the executive constructs and mathematical outcomes (Friso-van den Bos et al., 2013; Peng et al., 2016), and the separability of these constructs has been challenged (Miyake et al., 2000), especially in children (Best & Miller, 2010; Lerner & Lonigan, 2014). Thus any specific role for inhibitory control must be established over and above working memory capacity and to a lesser extent cognitive flexibility, yet most studies do not measure more than one executive function. In our own work, my colleagues and I have addressed these issues by employing tasks which capture semantic, rather than response inhibition, and collected multiple executive function measures. Specifically, we have used the color-word Stroop task (Stroop, 1992) where participants must report the font color of a written color word, which can either be congruent ("green" in green ink) or incongruent ("green" in red ink). Visual spatial working memory was assessed using an adaptive computerized version of the Corsi block-tapping task. Finally, we assessed cognitive flexibility with a task-switching measure where participants were first cued to report either the color or shape of a subsequent stimulus. In the subsequent probe phase, a colored shape is presented (e.g., red triangle) and participants then reported the appropriate dimension based on the preceding cue.

In terms of measuring of rational number understanding, we have focused our efforts on decimal numbers. We targeted the Category 2 property of *more digits, larger number*, based on the robustness of the effect (Huber et al., 2014; Varma & Karl, 2013). We found that performance on incongruent problems was correlated with color-word Stroop performance, as was working memory but not the cognitive flexibility task (Coulanges et al., 2021). Moreover, there were no correlations with problems which were consistent with prior whole number knowledge. With respect to relations with math achievement, all three executive function measures predicted performance on the Calculation subtest of the Woodcock-Johnson (Woodcock, McGrew, & Mather, 2001), but only inhibitory control was an independent predictor. Finally, we found that performance on the inconsistent decimal trials mediated the relationship between inhibitory control and math achievement, suggesting that greater inhibitory control may support understanding of counterintuitive rational numbers concepts, which may further support future math content learning.

In summary, the small set of studies using individual differences designs suggest that inhibitory control (especially, as measured by domain general semantic inhibition tasks) contributes to successful counterintuitive rational number processing. However, work remains to establish the independence of this relationship relative to other components of executive functioning and to track the developmental trajectory of this contribution. Another open question is the effects of whole number understanding on rational number learning: Would strong whole number knowledge require more inhibitory control to overcome whole number bias? While no studies to date have examined this question using an independent measure of inhibitory control, Van Hoof, Verschaffel, and Van Dooren (2017) found that better performance in whole number symbolic comparison and number line estimation were related to incongruent rational number understanding. One hypothesis from this line of reasoning is that superficial understanding of whole numbers (Category 2 properties) may require more inhibitory control to overcome, whereas grasping deeper features whole numbers, like magnitude coding (Category 3 properties), may set up a foundation for understanding these properties in rational numbers.

Magnitude representations of symbolic rational number quantities

The evidence presented thus far suggests that successful rational number processing draws on inhibitory control capacities to suppress interference from whole number knowledge. However, it must be acknowledged that while interference effects are pervasive, in proficient adults the effects are modest, especially at the highest ability levels. For example, a study of expert mathematicians found accuracy was over 94% on all categories of fraction comparison (Obersteiner et al., 2013). Only reaction times revealed the effects of whole number congruency, with common numerator problems solved more slowly than common denominator problems. And even this difference only amounted to 13% longer reaction times (1796 ms vs 2048 ms).

If inhibitory control were the only factor in successful rational number processing, we might expect that the proficiency would be marked by a reduction in performance costs for incongruent problems. That is, learners would simply improve in their capacity to execute the opposite of the whole number response. An alternative characterization proposes that mastering rational numbers involves conceptual change, that is, developing new knowledge structures, which enable ideas not possible in the original structure (Werner, 1957). Carey (2011) has noted that rational number learning has the hallmarks of conceptual change, in that it is very difficult to learn, and is marked by discontinuity in knowledge, noting "[w]hether children can properly order fractions and decimals, how they justify their ordering, how they explain the role of each numeral in a fraction expressed x/y, whether they agree there are numbers between 0 and 1 and whether they believe that repeated division by 2 will ever yield 0 are all interrelated" (p. 119). This assertion contrasts with longitudinal studies which suggest that magnitude-based ordering of rational numbers typically precedes arithmetic understanding, while understanding the density of rational numbers—there are infinite rational numbers between any two numbers—is the last of these concepts to develop (Kainulainen et al., 2017; Van Hoof et al., 2018).

Regardless of whether rational number understanding builds gradually (see Vosniadou, this volume) or reflects a discontinuous change (Carey, 2011), growing evidence supports the contention that deep understanding of rational numbers involves building new knowledge structures. In this chapter, I focus on the structure of rational numbers based on their magnitudes for three reasons. First, doing so demonstrates an expansion of magnitude-based representation from whole numbers, thus mastering a crucial Category 3 property. Second, individual differences in this capacity have been linked to general math achievement, suggesting it is a foundational capacity which promotes further mathematics learning (Matthews et al., 2016; Siegler, Thompson, & Schneider, 2011). Finally, from a practical perspective, magnitude-based representation can be operationalized in terms of the *distance effect* and therefore can be probed rigorously in behavioral and neuroimaging experiments.

The behavioral hallmark of magnitude processing—the distance effect—describes the phenomenon of higher accuracy and faster reaction times for numerical comparisons that are far apart (2 vs 9) relative to close (8 vs 9) comparisons (Moyer & Landauer, 1967). Among whole numbers, distance effects are found in both symbolic and nonsymbolic comparison task, and crucially, performance on these tasks has been linked to mathematics achievement (Schneider et al., 2017). Thus, there has been considerable interest in establishing that the processing of symbolic rational numbers is distance-driven.

Like other multicomponent numbers (Nuerk, Moeller, Klein, Willmes, & Fischer, 2011), rational numbers afford multiple distance dimensions. Here, I define *rational distance* as the actual difference between two quantities, which contrasts with *whole distance,* the distance between potentially interfering whole number quantities within a rational number. In the case of fractions, at least two whole dimensions can be identified: (1) distance between the numerators, and (2) distance between the denominators. Fig. 1 plots the relationship between rational and whole distance for all the single-digit

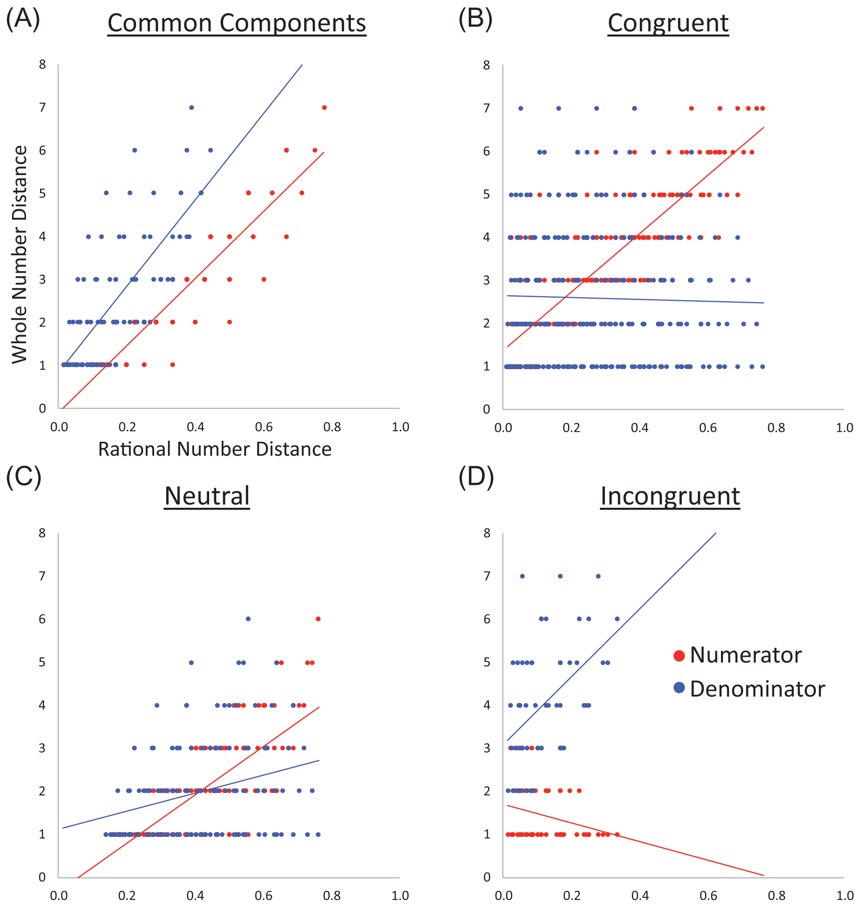

Fig. 1 Whole and rational number distance for all single-digit proper fractions. (A) *Common components*: same numerator problems (2/8 vs 2/3, rational distance = 0.42, denominator distance = 3, *blue*) and same denominator problems (3/7 vs 6/7, rational distance = 0.43, numerator distance = 3, *red*). (B) *Congruent*: larger numerator and larger denominator both indicate the larger fraction (1/7 vs 5/9 rational distance = 0.41, numerator distance = 4, denominator distance = 2). (C) *Neutral*: larger numerator and smaller denominator indicate the larger fraction (4/9 vs 6/7 rational distance = 0.41, numerator distance = 2, denominator distance = 2). (D) *Incongruent*: smaller numerator and smaller denominator both indicate the larger fraction (4/9 vs 3/4 rational distance = 0.31, numerator distance = 1, denominator distance = 5).

proper fraction comparison problems. As can be seen in the plot of common components problems (common numerators or common denominators, Fig. 1A), problems with comparable rational distance can vary on their whole distance. For example, 4/7 versus 4/5 and 1/3 versus 1/9 have a rational difference of 0.222 and 0.229 yet have denominator distance of 2 and

6, respectively. Thus if rational number magnitude is driving performance, these two problems should have comparable response times, whereas they should differ if whole distance is driving responses.

An early study, which tackled this question in adults, examined distance effects relative to a standard value (1/5, 0.2) for unit fractions (1/1, 1/2, 1/3, etc.) and in separate experiments, problems with common denominators (Bonato et al., 2007). Across four experiments, they found problems were solved componentially, or in our parlance, with respect to whole distance. Crucially, however, their experimental design never required a magnitude computation to reach the correct answer. For example, for unit fraction comparison, reversing whole number responses (i.e., selecting the smaller denominator in 1/2 vs 1/5) would always lead to the correct answer. Moreover, while the study reported for unit fractions that denominator distance better fit response times than rational distance, there was no acknowledgement that rational and whole distance correlates strongly for unit fractions. In fact, for all single-digit common numerator problems, there is a large correlation between these rational and denominator distance ($r(82)=0.746$), and similarly among unit fractions ($r(26)=0.742$). Even within the eight stimuli used in Bonato et al. (2007), this correlation is in the large range ($r(6)=0.598$).

Bonato et al. (2007) also considered the role of expertise, by contrasting psychology students with engineering, physics, and computer science students. However, their samples were small, ($n=\sim 10$ in each group) and gender was confounded with expertise. Regardless, they found a main of effect of expertise, but no difference in the reliance on whole distance. More recently, Obersteiner et al. (2013) examined a larger sample, with even greater expertise, mathematicians ($n=44$, 25% female, PhD students, postdocs and professors). They sampled stimuli from the set of 1–2 digit numerators and denominators fractions, in blocks with common components (either same denominator, e.g., 22/49 vs 18/49, or same numerator, e.g., 4/17 vs 4/39) or without common components, which could be congruent with whole number (larger fraction has larger numerator and larger denominator, e.g., 17/21 vs 9/14), incongruent (larger fraction has smaller numerator and smaller denominator, e.g., 11/25 vs 8/13), or neutral (larger fraction has larger numerator and smaller denominator, e.g., 7/16 vs 5/29). They found evidence that numerator, denominator, and rational distance all drove performance, in at least one of the conditions. Notably, the effects were strongest for rational distance for the problems without common components. The use of two-digit numerators and denominators affords more flexibility in stimulus selections. Yet, the pervasive correlations found in single-digit problems (Fig. 1) remain in this stimulus set. For example, among neutral problems, there are large correlations of rational distance with numerator ($r(16) = 0.549$) and denominator distance ($r(16) = 0.781$), and all three distance measures predicted reaction times among the mathematicians. Without orthogonalizing rational and whole distance, it is impossible to disambiguate the effects of each dimension and therefore to definitively assess the extent of magnitude-based representations of rational number.

Another factor complicating the interpretation of distance sensitivity in fraction comparison is the effects of strategy variation, which are especially prevalent in fraction comparisons (Obersteiner & Tumpek, 2016). As noted by Siegler (1999), efficient strategy choice is influenced by problem type, among other factors. For example, the incongruent problem 14/23 versus 5/6 has large numerator and denominator distances favoring the incorrect answer (9 and 17, respectively), yet if one compares the gaps between numerator and denominator (9 vs 1), it becomes clear that larger fraction (5/6) is also much closer to 1.

Strategies, like benchmarking (comparing to known values like 1/2) and comparing gaps, introduce new numerical dimensions, which in turn drive distance effects. For example, among the incongruent stimuli of Obersteiner et al. (2013), there is no meaningful correlations of rational distance with numerator ($r(16) = 0.034$) or denominator distance $r(16) = 0.169$). Yet, there is a large correlation with gap distance ($r(16) = 0.539$). In fact, gap distance will always lead to the correct response on incongruent problems, that is, the largest fraction always has the smaller gap (e.g., 8/11 vs 12/23, gap of 3 vs 11). In contrast, it can sometimes lead to the wrong answer for congruent problems (e.g., 17/39 vs 8/25, gap = 22 vs 17, yet, 17/39 is larger than 8/25). Although Obersteiner et al. (2013) did not explicitly test for the effects of gap distance, they did find unexpectedly slower responses on congruent relative to incongruent problems, and for incongruent problems, neither rational, nor numerator, nor denominator distance explained fraction reaction times among expert mathematicians. These results are all consistent with the possibility that gap distance drove performance for the mathematicians and the reported behavioral correlations with rational and whole distance may instead reflect stimulus correlations with gap distance. Gap-based strategies are a particularly fruitful area for future research because they may be a marker of more sophisticated relational understanding of fractions than componential strategies among learners who have yet to develop full-fledged magnitude-based representation of fractions.

To summarize these studies of fraction comparison in proficient adults, experimental evidence supports both whole and rational distance effects. However, insufficient attention has been paid to confounding factors within stimuli sets or to the role of strategy variation. Further compounding the effects of strategy variation are underappreciated contextual effects, which may bias participants toward or against certain representations. Presenting stimuli in blocks by type versus interspersing them may change the need to activate or suppress various magnitude representations, as in Bonato et al. (2007). In conclusion, for fractions it can be difficult, if not impossible, to design experiments that can definitively establish the independent effects of whole and rational distance.

In contrast, decimal comparison is much less studied and while some studies have examined magnitude-based processing (Kallai & Tzelgov, 2014; Zhang, Chen, Lin, & Szucs, 2014), to my knowledge, none have explicitly contrasted whole versus rational distance effects in this notation. Decimals are a ripe area for future research as the stimuli space affords greater opportunities for orthogonalizing distance dimensions. I have already highlighted the salient interference effect found in decimal numbers between the number of digits and numerical magnitude. Two explanations are given for this effect in the literature: *the string length congruity effect* proposes that participants independently compare individual place values and the number of digits in a string (Huber et al., 2014). In contrast, *the semantic interference effect* proposes that not just the number of digits, but the value of those digits influence comparison processes (Varma & Karl, 2013). For example, problems like 0.2 versus 0.12 and 0.9 versus 0.82 are both incongruent with whole number knowledge and had have the same rational distance of 0.08. Yet, if whole number knowledge is being processed (i.e., 2 vs 12, 9 vs 82), the first is much closer comparison than the second (10 vs 73), which may be further pushing participants toward the whole number response. Among congruent problems, comparisons like 0.1 versus 0.23 and 0.8 versus 0.93 should be equally difficult based on rational distance of 0.13, but based on whole distance the first is a much closer comparison than the second (22 vs 85).

Explicitly testing whether whole or rational distance is driving behavior requires carefully constructing the stimuli space. Fig. 2 plots all the combinations of tenths (0.1–0.9) versus hundredths (0.10–0.99) comparison problems with respect to whole and rational distance, organized by whole number congruency. As in fractions, there is a natural bias in the full space: among congruent problems, whole and rational distance are positively

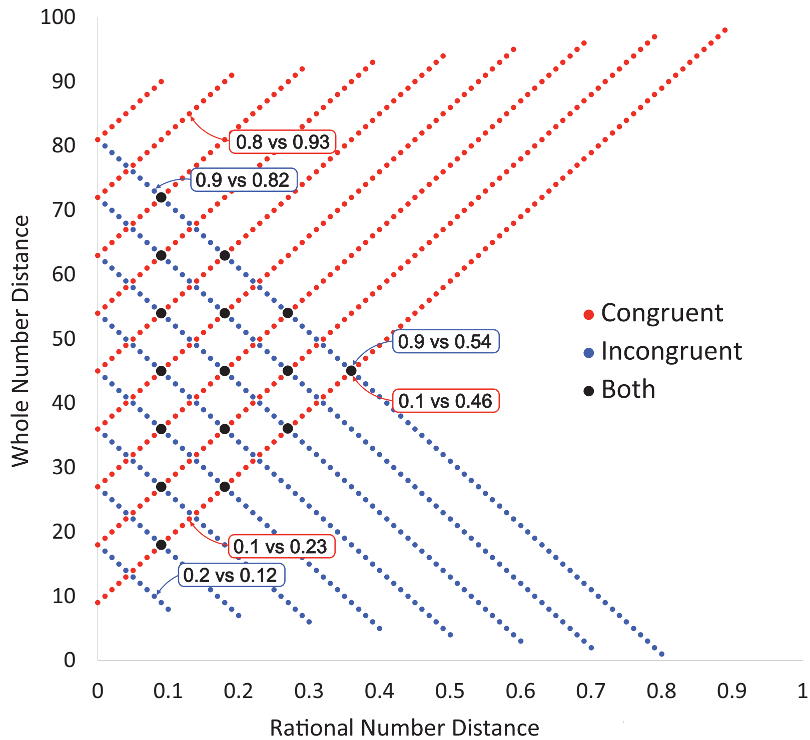

Fig. 2 Whole and rational number distance for all the combinations of tenths (0.1–0.9) vs hundredths (0.10–0.99) decimal comparison problems. In *red*: examples of congruent problems with identical rational distance (0.13), but differing whole distance (12 vs 85). In *blue*: examples of incongruent problems with identical rational distance (0.08), but differing whole distance (10 vs 73). In *black*: stimulus set with identical rational (e.g., 0.36) and whole (e.g., 45) distances for both congruent and incongruent problems. Among the *black* stimuli, whole and rational distance are matched across congruent and incongruent comparisons, and the distance dimensions are orthogonal.

correlated ($r(403) = 0.583$), while the correlation is negative among incongruent problems ($r(322) = -0.584$). In the latter case, that means that, on average, harder incongruent problems with smaller rational distances also have larger whole distances, further tipping the scales toward the whole number response.

My colleagues and I have controlled for rational distance across congruency conditions, in part by explicitly sampling near (rational distance: 0.16) and far problems (rational distance: 0.64) (Coulanges et al., 2021). However, by selecting problems with far rational distance, this approach confounds whole number distance with congruency (whole distance: congruent = 70.6; incongruent = 23.1). Thus to fully assess the impact of whole and rational distance on decimal processing, one must devise a stimulus set which orthogonalizes these properties and balances them over the condition types. Fortunately, within the full stimuli set is a region of overlapping whole and rational distance (Fig. 2), enabling just such an approach. In ongoing work, we have specifically selected problems with identical whole and rational distance for both congruent and incongruent stimuli (Pincus et al., 2020). This design can assess whether participants' responses are sensitive to whole distance while solving decimal comparison problems, and whether congruency effects are independent of the confounds of whole distance found in previous studies. Further, this design enables probing brain responses to whole versus rational distance, within the same individuals in a completely orthogonal design.

Nonsymbolic rational numbers and inhibitory control

The goal of studies of symbolic magnitude representation is to characterize knowledge states at various levels of instruction. In contrast, for nonsymbolic representations, work has focused on identifying precursor capacities that might predict later learning or be useful targets for intervention. To this end, researchers have focused on nonsymbolic, visuospatial representations of rational numbers, noting that individuals who perform better on these tasks also have better symbolic fraction understanding and math skills. For example, Siegler et al. (2011) found that better accuracy in placing fractions on a number line is related to higher math achievement. Another method of assessing magnitude understanding is comparison tasks, for example comparing two pairs of line lengths or sets of colored dots to determine the larger proportion. Using this task, Matthews et al. (2016) found that greater nonsymbolic comparison ability, which they term the ratio processing system (RPS), predicts symbolic understanding of rational numbers and performance on a college entrance exam.

These correlational studies provide tantalizing hints that instruction which emphasizes nonsymbolic representations could lead to improvements in rational number understanding. But to develop such curricula requires first selecting among the multiple possible nonsymbolic representations. Area models, the traditional approach to fraction instruction, involves dividing two-dimensional shapes into equal segments, like circles into wedges. This approach has been criticized because it often leads to misconceptions, like fractions can only be quantities less than one (Obersteiner et al., 2019). In contrast, the measurement approach, which divides objects along one axis, avoids some of these pitfalls and growing evidence supports the educational effectiveness of linking these nonsymbolic representations to symbolic fractions (Fuchs, Malone, Schumacher, Namkung, & Wang, 2017).

Here, I highlight another key distinction: continuous versus discrete representations. Both circles and line lengths can be presented continuously or discretized, while dots can only be discrete (Fig. 3A). While receiving less attention from the educational research community, a growing literature in cognitive development finds superior discrimination of continuous formats relative to discrete stimuli (Boyer, Levine, & Huttenlocher, 2008). For example, Jeong, Levine, and Huttenlocher (2007) found that children as young as six can select the larger proportion from two annuli with continuous sections. However, when those sections were divided into segments, performance dropped, specifically for stimuli where the larger proportion had fewer segments (see Fig. 3B). Further, while children's performance on continuous comparisons improved gradually from 6 to 10 years, performance on these "counting misleading" problems remained consistently poor over this same age range.

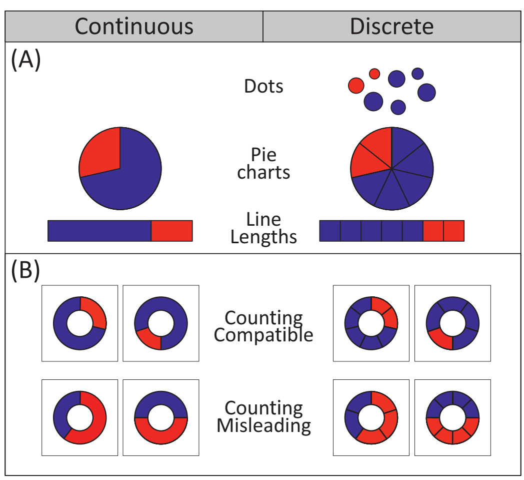

Fig. 3 Nonsymbolic stimuli format. (A) Sets of dots are always discrete stimuli, but pie charts and line lengths can be continuous, or segmented into discrete stimuli. (B) Sample stimuli from Jeong et al. (2007), adapted in Abreu-Mendoza, Coulanges, Ali, Powell, and Rosenberg-Lee (2020), contrast counting compatible stimuli (larger ratio has a larger number of discrete segments) with counting misleading stimuli (larger ratio has smaller number of discrete segments). These stimuli would be congruent (2/7 vs 1/5) and incongruent (3/5 vs 4/8) as symbolic fraction comparisons.

Difficulties with counting misleading discrete comparisons may be analogous to the congruency effects found in symbolic fraction comparison studies. In fact, the same stimuli that are incongruent in fraction comparison are counting misleading in segmented proportional reasoning (e.g., 3/5 vs 4/8, Fig. 3B). Given the relations between inhibitory control and incongruent symbolic rational number comparison (Avgerinou & Tolmie, 2019; Gómez et al., 2015), my colleagues and I reasoned that inhibitory control should also play a role in processing these misleading problems. In a study of second grade children (ages 7–8), we collected a computerized version of the Spinners task (Jeong et al., 2007) and the Hearts and Flowers task, a reliable measure of response inhibition for children as young as 3 years of age (Wright & Diamond, 2014). As expected, children with better inhibitory control also performed better on the counting misleading discrete trials (Abreu-Mendoza et al., 2020). Future work, in older children and adults, should examine relations between symbolic and nonsymbolic fraction performance and inhibitory control, to confirm this hypothesized connection between incongruent symbolic comparisons and their counting misleading nonsymbolic counterparts.

Another line of work in proportional reasoning examines improving discrete performance by priming participants with continuous information (Boyer & Levine, 2015; Hurst & Cordes, 2018). In our sample of second graders, we contrasted two groups of students: one saw continuous stimuli immediately before a discretized version of the same problems, while the other saw intermixed discretized stimuli (i.e., the segments of the same color were not adjacent). We hypothesized that seeing the continuous stimuli would prime proportional processing while the intermixed stimuli would activate counting strategies. Indeed, we found that immediately prior exposure to continuous information improved performance, specifically on the counting misleading trials (Abreu-Mendoza et al., 2020). Interestingly, among the continuously primed participants, the relationship with inhibitory control was reduced relative to the discretely primed participants. Although the interaction was not significant, these results are consistent with the proposal that subtle changes in context can bias comparison tasks towards or against the appropriate magnitude representations and thus impact the need for inhibitory control.

Brain imaging of rational number processing

The key contention of this chapter is that success with rational numbers requires refining and expanding one's representation of number. Support for the refining hypothesis would be provided by neural activity during incongruent rational number processing in brain regions identified with inhibitory control. For the expanding hypothesis, we would expect to see activity—especially magnitude-coding activity—for rational numbers in brain regions previously identified in whole number magnitude processing. Thus, this final section interrogates the small set of neuroimaging studies of rational number for evidence in support of these two predictions. Figs. 4 and 5 (Tables 2 and 3) present the reported parietal peaks from these studies

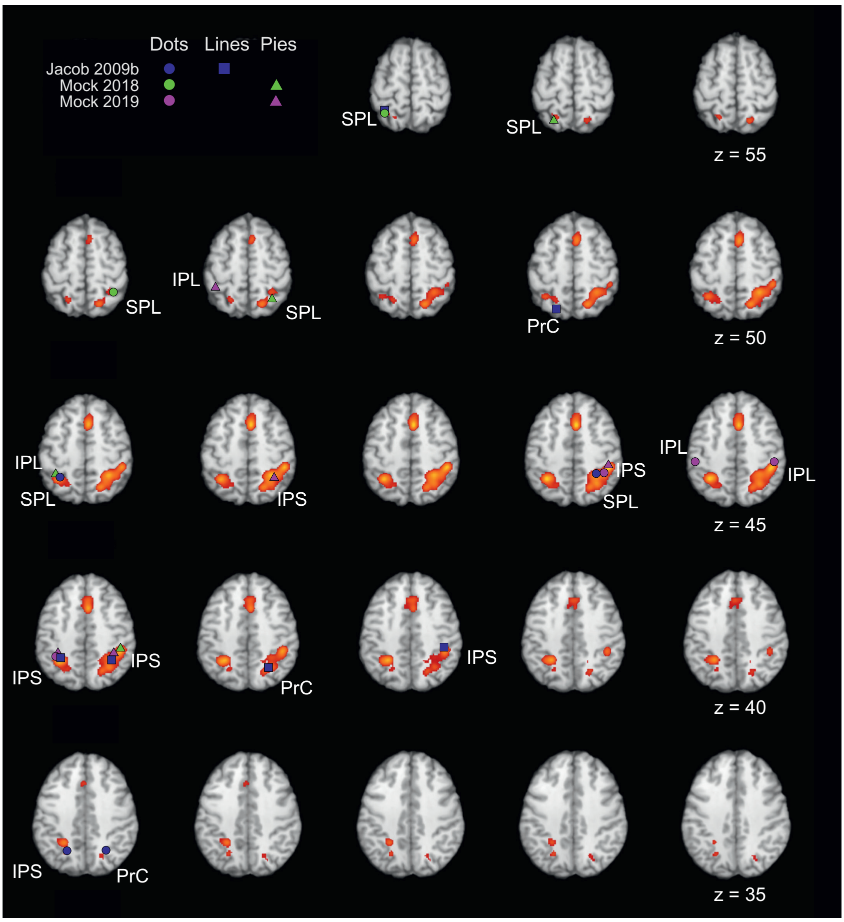

Fig. 4 Nonsymbolic parietal peaks for studies of rational numbers. ROIs: nonsymbolic peaks from Table 2; Functional clusters: nonsymbolic map (Sokolowski et al., 2017); Underlay brain: colin 1 mm^3, Talairach coordinates. IPS regions identified from the Juelich Histological Atlas, all other regions from the Talairach Daemon. *IPL*, inferior parietal lobule; *IPS*, intraparietal sulcus; *PrC*, precuneus; *SPL*, superior parietal lobule.

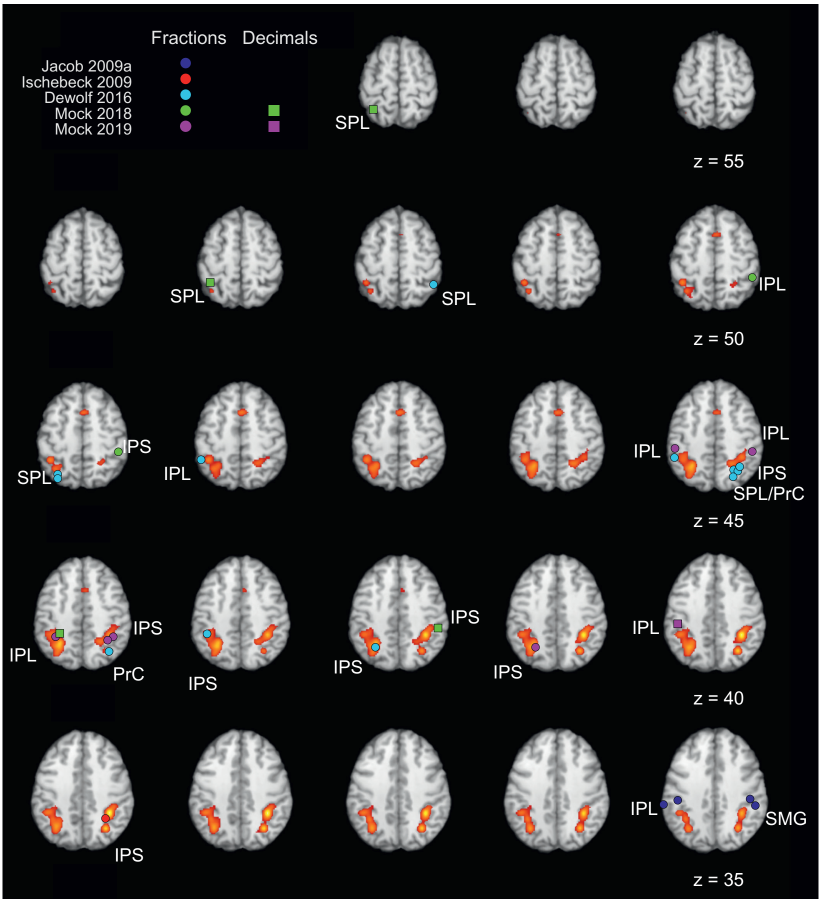

Fig. 5 Symbolic parietal peaks for studies of rational numbers. ROIs: symbolic peaks from Table 3; Functional clusters: symbolic map (Sokolowski et al., 2017); Underlay brain: colin 1 mm^3, Talairach Coordinates. IPS regions identified from the Juelich Histological Atlas, all other regions from the Talairach Daemon. *IPL*, inferior parietal lobule; *IPS*, intraparietal sulcus; *PrC*, precuneus; *SMG*, supramarginal gyrus; *SPL*, superior parietal lobule.

superimposed on recently published meta-analytic maps of nonsymbolic and symbolic whole number studies (Sokolowski, Fias, Mousa, & Ansari, 2017). Fig. 6 depicts the peaks from the only study to report the contrast of incongruent versus congruent fractions, overlaid on a "meta-analysis" of inhibition studies. The meta-analysis map was generated using Neurosynth.org, with the search term "inhibition," which identified 601 studies, using the forward inference test, at a false discovery rate of $P < .01$ (Yarkoni, Poldrack, Nichols,

Table 2 Nonsymbolic parietal peaks for studies of rational numbers (plotted in Fig. 4).

		MNI coordinates			Talairach coordinates		
Study	**Contrast**	**x**	**y**	**z**	**x**	**y**	**z**
Jacob and Nieder (2009b)	Dots (habituation)	24	−63	42	25	−59	39
		27	−51	51	28	−47	46
		−21	−63	42	−21	−60	39
		−30	−54	54	−30	−51	49
	Lines (habituation)	−30	−51	48	−30	−48	44
		−30	−54	63	−30	−50	57
		−18	−72	57	−18	−68	51
		24	−63	48	25	−59	43
		30	−54	48	31	−50	44
		39	−39	45	40	−36	42
Mock et al. (2018)	Dots (distance effect)	33	−52	60	34	−48	54
		−30	−57	63	−30	−53	57
	Pie charts (distance effect)	41	−40	48	42	−37	44
		28	−60	60	29	−56	53
		−22	−65	63	−22	−61	56
		−35	−50	53	−35	−47	49
Mock et al. (2019)	Dots > baseline (univariate contrast)	46	−35	48	46	−32	45
		36	−50	50	37	−46	46
		−47	−37	48	−47	−34	45
		−32	−50	48	−32	−47	44
	Pie charts > baseline (univariate contrast)	33	−45	48	34	−42	44
		41	−40	50	42	−36	46
		31	−55	53	32	−51	48
		−37	−45	58	−38	−42	53
		−32	−45	48	−32	−42	44

MNI coordinates were converted to Talairach coordinates (http://sprout022.sprout.yale.edu/mni2tal/mni2tal.html).

Van Essen, & Wager, 2011). The forward inference map, now called the uniformity test, represents the probability of finding activation in a voxel, given the search term was included in a study. This more liberal map was chosen, to maximally detect voxels identified with "inhibition," rather than the more conservative, reverse inference map (now called association test), which depicts the probability of a study mentioning the search term, given there is activity in that region.

Table 3 Symbolic parietal peaks for studies of rational numbers (plotted in Fig. 5).

		MNI coordinates			Talairach coordinates		
Study	**Contrast**	**x**	**y**	**z**	**x**	**y**	**z**
Jacob and Nieder (2009b)[a]	Fraction numerals (distance effect)	−62	−38	37	−61	−35	35
		51	−38	37	51	−35	35
	Fraction words (distance effect)	43	−30	37	43	−28	35
		−45	−31	37	−44	−29	35
Ischebeck et al. (2009)	Fraction numerals (distance effect)	27	−54	42	28	−51	39
DeWolf et al. (2016)	Fraction > integers (univariate contrast)	−28	−62	46	−28	−59	42
		−30	−74	54	−30	−69	49
		−40	−46	46	−40	−43	43
		30	−60	50	31	−56	45
		−30	−70	54	−30	−66	49
		−48	−50	52	−48	−47	48
		−48	−48	48	−48	−45	45
		32	−68	48	33	−64	44
		28	−64	50	29	−60	45
		24	−68	48	25	−60	45
		42	−54	58	43	−50	52
		22	−72	50	23	−68	45
Mock et al. (2018)	Fraction numerals (distance effect)	43	−42	53	44	−38	49
		46	−45	55	47	−41	50
	Decimals (distance effect)	48	−40	45	48	−37	42
		−37	−52	58	−37	−48	53
		−30	−57	63	−30	−53	57
		−27	−45	48	−27	−42	44
		−60	−45	30	−59	−43	30[b]
Mock et al. (2019)	Fractions > baseline (univariate contrast)	−47	−37	48	−47	−34	45
		−32	−50	48	−32	−47	44
		−25	−62	45	−25	−59	41
		46	−40	48	47	−37	45
		36	−50	48	37	−47	44
		28	−55	48	29	−51	44
	Decimals > baseline (univariate contrast)	−45	−32	43	−44	−30	40

[a] Peaks from Jacob and Nieder (2009a) are not reported in the paper, but are plotted with z-coordinate values in Fig. 2C (Fraction numerals) and Fig. 3B (Fraction words). These figures were used to approximate MNI co-ordinates. MNI coordinates were converted to Talairach coordinates (http://sprout022.sprout.yale.edu/mni2tal/mni2tal.html).
[b] Not plotted as outside the range of the z coordinates of the other ROIs.

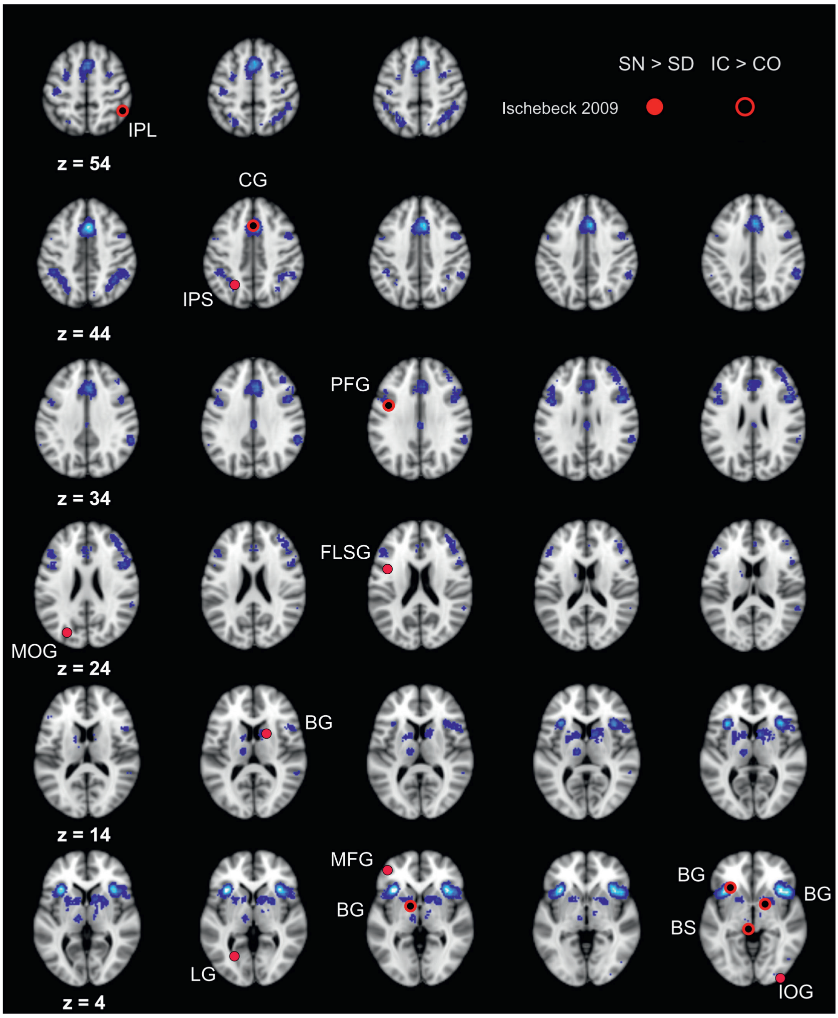

Fig. 6 Incongruency with whole number knowledge, whole brain peaks for studies of rational numbers. ROIs: incongruency contrasts of Ischebeck et al. (2009), peaks from Table 4; Functional clusters: neurosynth meta-analysis map of "Inhibition" (601 studies); Underlay brain: Mango Sample Image, Montreal Neurological Institute Coordinates. SN > SD = Same Numerator > Same Denominator, IC > CO = Incongruent > Congruent. IPS regions identified from the Juelich Histological Atlas, all other regions from the Talairach Daemon. *BG*, basal ganglia; *BS*, brainstem; *CG*, cingulated gyrus; *FLSG*, frontal lobe sub-gyral; *IOG*, inferior occipital gyrus; *IPS*, intraparietal sulcus; *LG*, lingual gyrus; *MFG*, medial frontal gyrus; *MOG*, medial occipital gyrus; *PFG*, precentral gyrus.

Considerable evidence points to the intraparietal sulcus as the key locus of whole number magnitude processing, both for symbolic and nonsymbolic stimuli (Arsalidou & Taylor, 2011; Sokolowski et al., 2017). The neural signature of magnitude process is sensitivity to the numerical distance between items, that is, greater activity for *near* (8 vs 9) than *far* comparisons (2 vs 9). Crucially, these effects are found in active comparison tasks, but also in passive viewing adaptation designs (Piazza, Pinel, Le Bihan, & Dehaene, 2007; Sokolowski et al., 2017), indicating that task difficulty alone does not drive neural distance effects. Developmental studies find that sensitivity to nonsymbolic distance is present early (Ansari & Dhital, 2006; Cantlon, Brannon, Carter, & Pelphrey, 2006) but develops slowly for symbolic stimuli (Ansari, Garcia, Lucas, Hamon, & Dhital, 2005; Bugden, Price, McLean, & Ansari, 2012; Rosenberg-Lee, Adair, & Menon, 2014). This pattern of results is consistent with the view that the ability of arbitrary number symbols to evoke magnitude representations in the IPS develops slowly with experience (Rosenberg-Lee, Tsang, & Menon, 2009) and suggests that accurate representations of rational number magnitudes should also engage the IPS in proficient adults.

This prediction is borne out in the handful of studies that have examined brain responses to rational numbers. Jacob and Nieder (2009b) used a passive viewing design where participants saw multiple instances of the same ratio (e.g., adapting to 1:5) represented either as pairs of unsegmented lines or sets of colored dots and then were presented with a deviant ratio (e.g., 4:5). They found that the recovery of BOLD signal scaled with the distance from the adapted ratio, in both the left and right IPS (Fig. 4). To assess the overlap with regions involved in nonsymbolic whole number processing, they also collected a dots adaptation task, finding surprisingly consistent activity between the two formats in their adult participants. A second study from this group used this same framework to examine symbolic processing, both of numerals and of fraction terms like "half" (Jacob & Nieder, 2009a). Again, they reported activity in the IPS which was sensitive to the numerical distance between the adapted and deviant stimuli, now in the symbolic domain (Fig. 5). Adaptation studies of this type are useful because they demonstrate that brain responses do not simply reflect behavioral difficulty effects but also abstract representation (Binder, Desai, Graves, & Conant, 2009). Further, in the current context, they demonstrate that the same regions employed for whole numbers are used for symbolic fraction and nonsymbolic symbolic ratios. However, neither study contrasted congruent and incongruent stimuli and so do not provide information on the brain loci needed to resolve whole number interference.

Only one study to date has attempted to identify brain regions sensitive to both rational and whole distance within a single task and sample ($n = 17$, Ischebeck et al., 2009). Using single-digit proper and improper fractions, they presented stimuli across four conditions. The common components' problems (i.e., same numerators, same denominators) followed typical practices in the field. However, for without common components problems, they assumed that individuals know that larger denominators indicate smaller numbers, so they considered problems such as 2/7 versus 3/6 as a congruent and 2/6 versus 3/7 as an incongruent, while the standard nomenclature would have designated them neutral and congruent, respectively (Fig. 1). Although their behavioral results were consistent with their designation (participants were slower on incongruent relative to congruent problems), it calls to question if these stimuli were tapping whole number interference, per se. Finally, rational distance varied continuously in their stimuli, but they explicitly manipulate whole distance into a two-level factor by making numerators or denominators 1 or 3 units apart, while keeping the other value the same (common components) or differing only by 1 (without common components).

Interestingly, whole distance drove accuracy, while rational distance explained response times, especially on their "incongruent" problems. Most crucially, with respect to brain activity, the IPS was sensitive to rational distance regardless of condition and no brain region was related to whole distance (Fig. 5). This is the only study to report neural congruency effects, finding greater activity for common numerator relative to common denominator comparisons, and incongruent relative to congruent comparisons, in regions which largerly overlapped those involved in inhibitory control, including medial and lateral prefrontal regions. Fig. 6 (Table 4) plots these locations with respect to "inhibition" studies identified by Neurosynth.org (Yarkoni et al., 2011).

More recently, DeWolf et al. (2016) applied representational similarity analysis (RSA) to examine relations between activity patterns for whole numbers and rational numbers represented as fractions or decimals. Stimuli were single-digit fractions, the equivalent values as decimals, rounded to 2 decimal points and the corresponding whole number quantities. For example, the fractions 1/9 versus 3/7 correspond to 0.11 versus 0.43 as decimals and 11 versus 43 as whole numbers. The authors employed a sequential presentation order and only analyzed the brain responses from the first stimuli. This approach is ideal for RSA because each number is presented in isolation and so its brain activity can be assessed without the potential confounding effects of processing the other number, or the brain responses to

Table 4 Incongruency with whole number knowledge, wholebrain peaks for studies of rational numbers (plotted in Fig. 6).

Study	Contrast	MNI coordinates		
		x	y	z
Ischebeck et al. (2009)	Same Numerator > Same Denominator[a]	− 42	6	21
		− 42	48	0
		18	9	12
		− 24	− 57	42
		36	− 93	− 3
		− 27	− 78	24
		− 24	− 66	3
	Incongruent > Congruent	− 42	0	30
		0	21	42
		− 30	24	− 3
		39	21	− 6
		− 12	0	0
		15	3	− 3
		− 6	− 30	− 3
		48	− 48	54

[a] In Table 3 of Ischebeck et al. (2009), this contrast is labeled as SD > SN (Same Denominator > Same Numerator). However, the text and Fig. 3 describe the contrast as Same Numerator > Same Denominator, thus we are using that label and interpretation here.

the comparison action (Dimsdale-Zucker & Ranganath, 2018). However, sequential analyses preclude assessment of numerical distance effects in brain responses.

RSA is a multivariate neuroimaging data analysis technique that involves extracting voxels for different experimental conditions and computing the similarity (e.g., Pearson correlation) between them (Kriegeskorte, Mur, & Bandettini, 2008). Using a searchlight approach over an IPS region of interest, Dewolf and colleagues found that activation patterns for fractions (e.g., 1/3) differed from decimal (e.g., 0.33) and whole (33) number patterns, which were indistinguishable from each other (DeWolf et al., 2016). Univariate analyses were convergent, finding greater activity for fractions in the IPS than for decimals or whole numbers. While these results confirmed that fractions are processed in the IPS, the experimental design of the decimals stimuli complicate interpretation of their results. Specifically, since the decimal stimuli were always presented with two significant digits, there is no need to process these stimuli with respect to their rational number magnitude, likely accounting for the high similarity between decimal and whole numbers.

To determine if magnitude processing regions are shared across rational number formats, Mock et al. (2018) examined two formats of symbolic (fractions and decimals) and nonsymbolic (unsegmented pie charts and dot sets) rational number representation. Base stimuli were created from single-digit fractions with numerators ranging from 1 to 8 and denominators from 2 to 9. While they manipulated congruency in the fraction stimuli using field typical labels (Fig. 1), they did not report behavioral or imaging contrasts based on this distinction. As in DeWolf et al. (2016), decimal stimuli were created by converting to decimals and truncating at two significant digits. Although the authors did not characterize their stimuli in this fashion, their nonsymbolic stimuli of pie charts and dot proportions end up creating a design which contrasts continuous versus discrete representations. Specifically, in the dot proportions stimuli, the incongruent stimuli would also be considered as counting incompatible as conceptualized in Jeong et al. (2007). Unfortunately, this factor was not contrasted in either their behavioral or brain imaging results. However, it's notable that the two factors which contained incongruent stimuli (fractions and dot proportions) also had the slowest speeds and lowest accuracies of the four formats.

In the imaging analyses, they sought brain regions sensitive to the rational distance between the numerical pairs. In a conjunction analysis, they found clusters in the right IPS sensitive to rational distance. While this result is consistent with the proposal that the IPS is expanded to process rational distance, because they do not orthogonalize whole distance, it's impossible to determine if these results simply reflect the whole distance, which would be driving all the activity, in, for example, the decimals conditions. Consistent with this contention are results from a follow up study from the same group using the same data (Mock et al., 2019). There, they found greater activity for the tasks which they say measure part-whole relationships (fractions, dot proportions, and pie charts) relative to decimals in bilateral IPS and frontal regions. They also compute region of interest-based RSA, finding greater similarity between dots and fractions in both the left and right IPS, while decimals and fractions were least similar. The authors interpreted this result to mean that part-whole relations have different activation patterns than decimals. Alternatively, both fractions and dots proportions draw on inhibitory control because of interference from whole number knowledge leading to similar activation patterns, whereas these specific decimals and continuous pie charts lacked these demands.

In summary, neuroimaging studies of rational numbers find activity and especially distance-based activity (Figs. 4 and 5) in the bilateral IPS and nearby parietal regions that have previously been associated with symbolic and nonsymbolic number processing (Sokolowski et al., 2017). However, insufficient attention has been paid to assessing rational distance effects independently from the confounding influence of whole distance. Moreover, while congruency with whole number knowledge is often manipulated in imaging studies, the contrast is rarely reported for symbolic studies, and never for nonsymbolic studies (Fig. 6). Therefore, the paltry evidence regarding the overlap of inhibitory control systems with regions involved in processing whole number interfering rational numbers means we cannot draw firm conclusions from imaging studies about the role of inhibitory control regions in rational number processing.

Conclusions and directions for future research

In this chapter, I have considered less studied factors that make rational numbers difficult for many learners. First, I distinguished between properties of whole numbers that never apply to rationals (Category 1) from properties that sometimes, but not always, apply (Category 2). Implicit learning of the whole number properties may further exacerbate difficulties with Category 2 properties. Future research, probing the effects of explicitly highlighting the variable nature of these properties, could assess the effects of implicit understanding on rational number outcomes. A final category of properties (Category 3) does, in fact, apply to rationals, but learners struggle to extend them to this new number system. Thus, I contend that mastery of rational numbers requires refining and expanding one's numerical representation. In terms of refining, while many studies report whole number interference effects, only a handful explicitly test whether interference effects are related to individual differences in inhibitory control. Measuring inhibitory control is challenging, especially across development. I advocate for tasks which tap "semantic inhibition"; empirically these tasks show stronger relations with rational number outcomes compared to response inhibition tasks. Further, from a theoretical perspective, interference effects in rational numbers come from acquired understanding of number systems and thus should draw inhibition of semantic knowledge.

To assess expanding numerical representations, studies contrasting rational and whole distance were reviewed. For fractions, I highlight the inherent correlations between these dimensions (Fig. 1). and suggest that strategy variation further complicates matters, as factors like gap distance may also drive responses, but only among participants sophisticated enough to use this strategy. In contrast, I propose that decimals provide an ideal domain for contrasting rational and whole distance. Specifically, the property of *more digits = larger number* represents a Category 2 property with a well-defined behavioral effect, reducing the potential for strategy variation. From a practical perspective, decimals have only one whole number dimension (versus at least two dimensions for fractions) and there exists a stimuli set that equates whole and rational distance between congruent and incongruent problems, while keeping these factors orthogonal (Fig. 2). This set provides a platform for behavioral and imaging studies to establish the developmental trajectory of magnitude-based processing of rational numbers. In the domain of nonsymbolic representations of rational quantities, I highlight the distinction between continuous and discrete formats (Fig. 3). Further, nonsymbolic counting misleading discrete problems can be connected to symbolic incongruent fraction problems, suggesting that future work should examine individual differences in these effects and relate them to inhibitory control.

Finally, turning to neuroimaging studies, which we evaluated in the context of these theoretical and methodological issues. Notably, while many studies manipulate the congruence of rational number stimuli, only one explicitly reports the neural contrast (Ischebeck et al., 2009). Thus we know almost nothing about how brain regions involved in inhibitory control contribute to successful rational number processing. In terms of identifying distance-based processing of rational numbers, there is some evidence that these effects are found in the IPS, overlapping with, or near to, regions identified in a meta-analysis of whole number processing (Sokolowski et al., 2017). Notably, no studies have examined interference from whole number knowledge in decimals, and studies of symbolic fractions have not adequately addressed confounds across problem types. In terms of nonsymbolic formats, inadequate attention has been paid to the role of continuous versus discrete formats. Representational similarity analyses are an important tool in deciphering neural representations of rational numbers. The finding of greater similarity of fractions to discrete dots relative to continuous pie charts (Mock et al., 2019) supports the contention that whole number interference effects may be driving both impairments on discrete nonsymbolic comparisons and symbolic fractions. Another neuroimaging methodology, event-related potentials may be especially fruitful in this domain, as considerable work has already mapped the neural signatures of conflict processing in both the numerical (Szucs & Soltesz, 2007) and color-word (Atchley, Klee, & Oken, 2017) Stroop tasks, and a growing literature is now examining these signals in fraction comparison (Fu, Li, Xu, & Zeng, 2020; Zhang et al., 2012).

Missing from the imaging literature are examinations of the relations between individual variability in ability and brain responses: Do individuals with greater whole number interference show less magnitude processing for rational numbers? Finally, to date, there are no published imaging studies of rational number processing in children. It is vital to track the expansion and refining of number representations through the acquisition of this challenging material. If we can identify the end state of successful math instruction, then we can combine instruction and neuroimaging to track the efficacy of different instructional approaches in inducing the desired proficient performance profile in both brain and behavior (Rosenberg-Lee, 2018).

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